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Getting to grips with hadrons

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Abstract. A short discussion concerning the lattice QCD approach to physics of hadrons is made to non-specialists. A special attention is given to topics that are of particular interest to the nuclear physics community.

PACS. 11.15.Ha Lattice Gauge Theory – 12.39.Fe Chiral Lagrangians

1 Why lattice QCD?

One of the most important questions that still remains to be answered is to explain how hadrons arise from the QCD lagrangian

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{q=u,d,s,\dots} \bar{q}\left\{\gamma_\mu\left(\partial_\mu + g A^a_\mu t^a\right) + m_q\right\} q \,. \quad (1)$$

The dynamics that governs the confinement of quarks and gluons into hadrons is of notoriously nonperturbative nature for which an analytic treatment is still missing. Although various quark models help understanding quite a number of phenomena of hadronic interactions, it should be stressed that a covariant quark model that solves simultaneously confinement and spontaneous chiral symmetry breaking has never been constructed. Other than quark models, much effort has been put in building the effective theories of QCD, valid for specific ranges of low energy scale. Those theories are built upon some symmetry property of the QCD lagrangian in some specific limit. The most prominent example is the chiral (left↔right) symmetry, $SU(N_f)_L \otimes SU(N_f)_R$, that is manifest when the quarks are massless. That symmetry is spontaneously broken down to $SU(N_f)_V$, resulting in the appearance of $N_f^2 - 1$ Goldstone bosons ('pions'). Chiral perturbation theory (ChPT) provides us with an effective description of QCD that incorporates these features and, in addition, allows one to account for the explicit chiral symmetry breaking corrections, namely those generated by the non-zero quark mass terms in the QCD lagrangian. The computation of such corrections, unfortunately, generates a bunch of low energy constants that are supposed to be obtained from the matching procedure of appropriately chosen amplitudes computed both in ChPT and in QCD, at some energy scale at which ChPT can be trusted and at which direct QCD computations can be made. This is where lattice QCD is expected to provide information to the QCD piece in this matching. In the above discussion N_f stands for the number of light quark flavors. Today we are confident that ChPT provides an adequate framework to describe the dynamics of strange-less hadrons ($N_f = 2$), whereas the situation with the strange quark (m_s) is still unclear [1]. This is not only because m_s is about 25 times larger than $m_q = (m_u + m_d)/2m_u$ [2], but also because it is not much smaller than the hadronic QCD scale, $\Lambda_{\rm QCD}^{\overline{\rm MS}\,(N_f=3)} = 336^{+42}_{-38}$ MeV [3]. What do we know about m_s? This is one of the highlights of the lattice QCD achievements over the past decade which is why I decided to briefly discuss it here. That discussion will also allow me to introduce the methodology but also the challenges of lattice QCD.

1.1 Lattice QCD and the strange quark mass

The numerical solution to the problem in hands, namely to compute the hadronic spectra numerically from the QCD lagrangian only (1), does exist. The crucial first step in that direction is to make the analytic continuation to the euclidean metric ($x_0^M \to i x_0^E$), in which the QCD generating functional reads

$$Z[A,q,\bar{q}] = \int \mathcal{D}A\mathcal{D}q\mathcal{D}\bar{q}\, \exp\{-S[A,q,\bar{q}]\} \,. \quad (2)$$

In euclidean space the QCD action is real and bounded from below. Discretization of the euclidean space and time, $L = N_S a$ and $T = N_t a$, allows for an equivalence between the generating functional and the partition function, so that the Monte Carlo methods can be employed. $SU(3)$ gauge fields are placed on the links of the lattice whereas the quark degrees of freedom are sitting on the sites. A particularly important feature, while discretizing the QCD action, is that the gauge invariance is preserved at every stage of calculation. The price to pay is that the lattice spacing $a \neq 0$ breaks the Lorentz invariance, which is however recovered once we take the continuum limit, $a \to 0$ (i.e. after we send the UV cut-off to infinity). Finally, after the continuum limit has been taken appropriately, we should worry about the finite volume effects and work out the limit $L, T \to \infty$ (i.e. send the IR regulator to zero). This is a very challenging task for numerical simulations, and it requires a lot of clever ideas and a huge computing power. What is important to retain is that –in principle– the QCD simulations on the lattice offer a first principle approach to the physics of hadrons. In other words the solution to nonperturbative QCD is provided without introducing any additional parameter except for those that appear in the QCD lagrangian, namely the quark masses and the $SU(3)_c$ gauge coupling. In practice, however, various approximations are often necessary in order to make the calculation feasible on the present day computing resources. Importantly though, all those approximations are controllable and, for the most part, we can get rid of them by increasing the computing power. The most infamous (least controllable) is the so-called quenched approximation. It consists in neglecting the dynamical quark loops while producing the gauge field configuration. This is certainly a serious drawback of the most of currently available lattice results, but it nevertheless make a good case for developing the methodology for the computation of various physical quantities on the lattice. One way to compute the quark mass on the lattice is via the axial Ward identity [1], $\partial_\mu A_\mu(x) = 2m_q P_5(x)$. One computes the following two correlation functions:

$$\langle \sum_{\mathbf{x}} \partial_\mu \underbrace{\bar{q}(x)\gamma_\mu\gamma_5 q(x)}_{A_\mu(x)} \mathcal{O}(0)\rangle \quad \text{and} \quad \langle \sum_{\mathbf{x}} \underbrace{\bar{q}(x)\gamma_5 q(x)}_{P_5(x)} \mathcal{O}(0)\rangle ,$$

where $\mathcal{O}$ is a bilinear quark operator with quantum numbers $J^P = 0^-$, and after having properly renormalized the axial current and the pseudoscalar density, the ratio of these two correlation functions gives the quark mass. Various ways to nonperturbatively renormalize the composite quark operators on the lattice have been developed (see [4]) and they are implemented in most of the present day quark mass calculations. Besides the ratio of the above correlation functions, from the exponential dependence of the second correlator one can read off the corresponding pseudoscalar meson mass. At this point it should be stressed that the lattice results are consistent with the Gell-Mann–Oakes–Renner (GMOR) formula, $m_{PS}^2 = 2B_0 m_q$. Surprisingly, however, although the GMOR formula is expected to be valid for very small quark masses (it receives the m_q^2-corrections and higher), the lattice QCD results (with Wilson fermions) display a rather impressive consistency with the leading GMOR formula while working with heavy pions ($m_{PS}^2 \geq 500$ MeV). The strategy to reach the physical quark mass is to tune the quark mass in the QCD action in such a way that the corresponding pseudoscalar meson mass coincides with the physical kaon mass. The resulting strange quark mass by various lattice collaborations have been obtained by means of high statistics simulations, by implementing the non-perturbative renormalization on fine grained lattices, so that the continuum limit could be taken. Finite volume effects have also been examined and shown to be tiny, i.e., at the level much smaller than the errors they quote. The results, listed in Table 1, are obtained in the quenched approximation. Important qualitative outcome from the lattice studies is that the quark masses are indeed small and that the light hadron masses are mostly due to QCD interaction rather than to their valence quark content. Finally notice that the first lattice studies in which the effects of dynamical quarks are included show that the strange quark mass gets even smaller [7], but we are not yet at the stage of providing the precision unquenched computation of m_s.

[1] For alternative strategies and lattice actions to compute the strange quark mass, please see [5].

Table 1. Strange quark mass obtained from the quenched QCD simulations on the lattice. Results by various collaborations [6] refer to the continuum limit ($a \to 0$)

collaboration	$m_s^{\overline{\rm MS}}(2\text{ GeV})$
JLQCD	106(7) MeV
Alpha & UKQCD	97(4) MeV
QCDSF	105(4) MeV
CP-PACS	111^{+3}_{-4} MeV
SPQcdR	106(2)(8) MeV

2 Hadron spectrum

Lattice QCD is particularly well suited to study the spectra of hadrons. In the previous section we already mentioned that the pseudoscalar meson masses were necessary to identify the strange quark mass. One can also study the correlation functions with the interpolating bilinear quark operators carrying other quantum numbers and thus extract the vector, axial-vector, tensor and even scalar mesons (for the last the valence quarks should be non-degenerate in order to have the correlator with a discernible signal).

2.1 Glueballs

In spite of the quenched approximation, some long standing problems can still be tackled. One such a problem is the existence of the mass gap in the pure Yang–Mills theory. This problem is stated as one of Seven Millenium Math Prize Problems [9], to which an analytical solution is missing. On the other hand, many lattice analyses performed so far show that the glueball states indeed exist. Nowadays even the spectrum of such states has been established. This is a very important prediction of lattice QCD. The spectrum shown in Fig. 1, is given in multiples of r_0, a quantity that is defined from the condition $r \cdot dV(r)/dr|_{r=r_0} = 1.65$ [10], where $V(r)$ is the potential

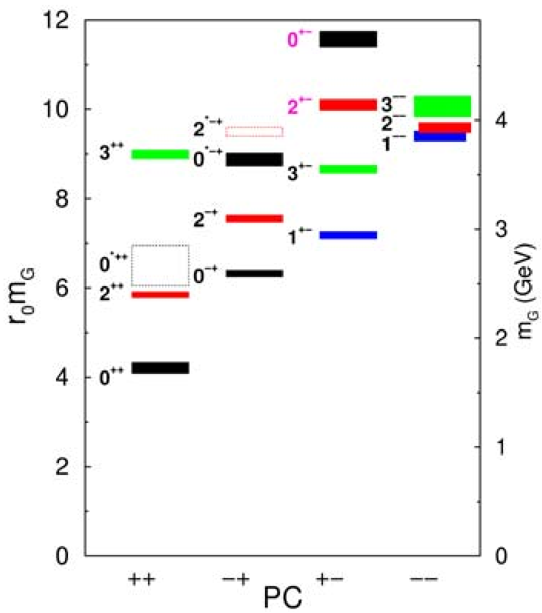

Fig. 1. The spectrum of glueball states as established from the extensive quenched lattice QCD simulations in [8]. The widths of the lines reflect the *error bars* of lattice results

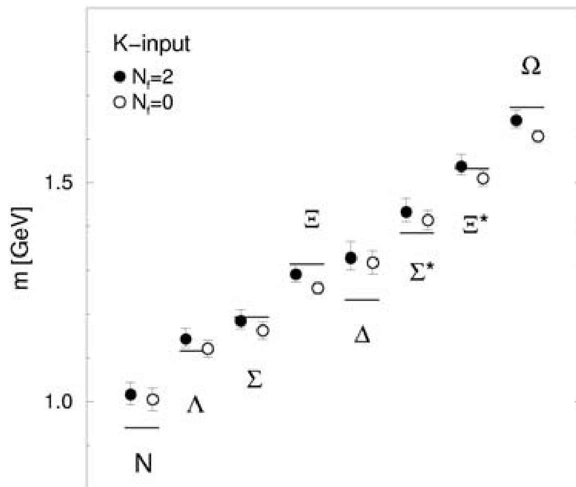

Fig. 2. The spectrum of baryons produced by JLQCD both in quenched ($N_f = 0$) and in unquenched ($N_f = 2$) QCD [11]. Physical (experimentally established) masses [12] are marked by the *horizontal lines*

between two infinitely heavy quarks, which can be (and has been) accurately studied on the lattice. To convert to physical units, a commonly assumed value is $r_0 = 0.5$ fm (or $r_0 = 2.5\ \mathrm{GeV}^{-1}$).

2.2 Baryons

As we already mentioned, from the exponential fall-off of various correlation functions (made with various interpolating operators consisting of quark and gluon fields), one can extract the hadron masses with quantum numbers carried by the considered composite operator. The operators used to extract the proton mass (and its coupling to these interpolating operator) are

$$\begin{aligned} J(x) &= \varepsilon^{abc}\left[u_a^T(x) C\gamma_5 d_b(x)\right] u_c(x), \\ \tilde{J}(x) &= \varepsilon^{abc}\left[u_a^T(x) C d_b(x)\right] \gamma_5 u_c(x), \end{aligned} \tag{3}$$

where C stands for the charge conjugation operator. Neutron mass is simply obtained by replacing one u quark by d, whereas the Ξ state arise after replacing u and d by two s quarks, and so on. The spectrum of lowest baryon states computed on the lattice is shown in Fig. 2. Strange quark mass is fixed from the physical kaon mass, as explained in the previous section. The most striking feature of that plot is that the baryon spectrum, as deduced from the quenched simulations is essentially unchanged after unquenching the QCD vacuum fluctuation by $N_f = 2$ dynamical quarks. This probably indicates that the most significant effect of quenching has been absorbed in the conversion of results from the lattice to physical units. Second important feature is the nucleon mass that in both cases is larger than that of the physical nucleon. To discuss the reasons for that effect we should stress that the nucleon mass is not obtained directly on the lattice but rather after a long extrapolation. This is so because the lattice simulations are performed with the light quarks $m_q \geq m_s^{\mathrm{phys}}/2$, with $m_q \equiv m_u = m_d$, while the physical limit is $m_q/m_s = 0.04$. Since the sector of light quark masses over which one has to extrapolate is expected to be highly sensitive to the effects of spontaneous chiral symmetry breaking, it is not enough to extrapolate the linear (or quadratic) quark mass dependence observed with the directly accessible quarks (i.e. in the 'heavy pion world'). Therefore, the task, that the lattice community approached very seriously, is to reduce the value of simulated quark ('pion') masses. The trouble is that reduction is very costly in computing power. Even if we manage to create clever algorithms to work closer to the chiral limit the artifacts due to finiteness of the lattice size (L) become more pronounced and the chiral extrapolations should be made by using the formulae derived by using the chiral perturbation theory in the finite volume. That issue attracted quite a bit of attention in the lattice community over the past couple of years [13]. In the case of nucleon, the leading order chiral lagrangian

$$\mathcal{L}_N^{(1)} = \bar{\Psi}\left(i\gamma_\mu D^\mu - m_0\right)\Psi + \frac{1}{2} g_A \bar{\Psi}\gamma_\mu\gamma_5\xi^\mu\Psi, \tag{4}$$

is consisted of the nucleon field Ψ, the covariant derivative $D_\mu = \partial_\mu + \frac{1}{2}[\xi^\dagger, \partial_\mu \xi]$, in which the Goldstone boson fields enter via $\Sigma = \xi^2$, and $\xi_\mu = i\xi^\dagger \partial_\mu \Sigma \xi^\dagger$. The standard axial coupling is used, $g_A = 1.2$. One-loop chiral corrections to the self energy of the nucleon propagator produce the shift to the bare nucleon mass, m_0, as

$$\begin{aligned} m_N = m_0 - 4c_1 m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 \\ + \left[C(\Lambda) - \frac{3g_A^2}{2m_0}\frac{1}{(4\pi f_\pi)^2}\left(1 + \log\frac{m_\pi^2}{\Lambda^2}\right)\right] m_\pi^4 + \ldots \end{aligned} \tag{5}$$

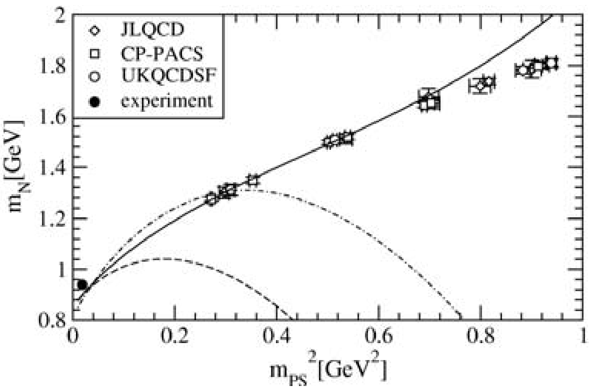

Fig. 3. The chiral extrapolation of the lattice QCD results with $N_f = 2$. *Solid* and *dashed curves* correspond to the so-called infrared regularization and to the non-relativistic treatment of the nucleon. For more information, please see [15]

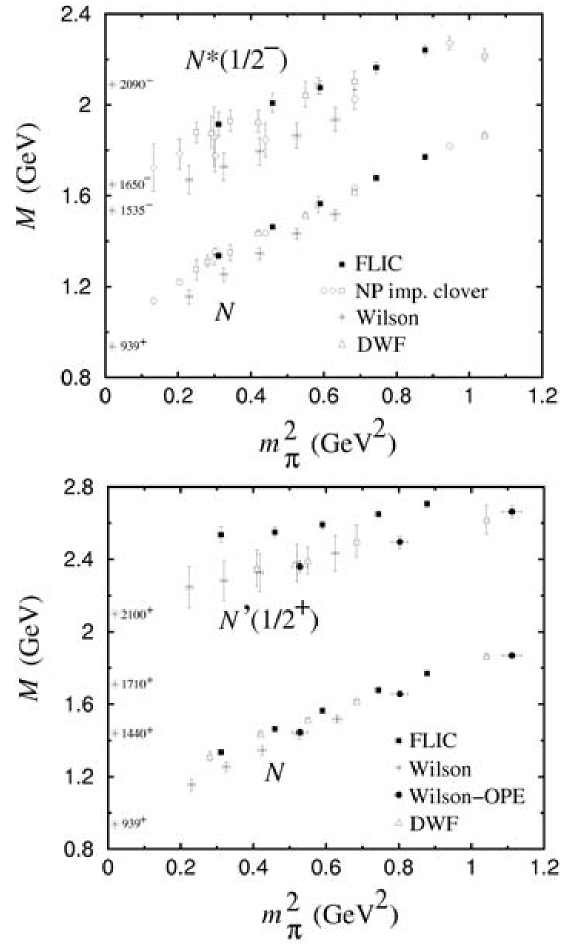

Fig. 4. Results for the orbital (*upper plot*) and radial (*lower plot*) excitations as a function of the pion mass directly accessible from the quenched QCD simulations on the lattice. For more details please see [21]

where $C(\Lambda)$ is the counter-term which cancels the Λ-dependence that otherwise arises from the renormalization of the ultraviolet divergences in the chiral loops. It turns out, however, that the two conventional descriptions lead to quite different results when applied to the lattice data with $N_f = 2$, and that they coincide only for very light pions, namely $m_\pi \ll m_N$ [14,15]. This is shown in Fig. 3 where the solid curve indicates the fit to lattice data and by using the expressions derived by means of the so-called infrared regularization [16]. The non-relativistic treatment [17] is depicted by the two dashed lines corresponding to two specific choices of parameters c_1 and m_0. Besides providing the guidelines to extrapolate to the physically interesting limit for nucleons, ChPT is also useful in estimating the impact of the effects of the finiteness of the lattice box. It verifies the general Lüscher formula [18] and provides the corrections to it. That highly useful aspect of ChPT has been extended to other quantities involving baryons [19]. Important to retain is that the chiral logarithmic behavior gets enhanced by the finiteness of the lattice box. This makes the chiral extrapolations ever more delicate: physical chiral logarithms are mostly drowned in the finite size effects, and therefore one does not only seek the range of quark masses in which the ChPT are valid description of the lattice data, but one also has to disentangle the finite size effects from physical chiral logarithms (former being often overwhelming compared to the size of the latter).

Finally, it should be noted that the above discussion refers to the so-called p-regime (i.e., with $m_\pi L \gg 1$). A probably viable alternative has been recently proposed in [20] where it is claimed that the ϵ-regime ($m_\pi L \ll 1$) might be helpful in discerning the finite volume behavior and thus extract the terms involving the low energy constants, such as $C(\Lambda)$ in (5). Clearly, whatever the regime is considered (p or ϵ), one should make sure that the ChPT formulas provide the fiducial description of the lattice data, and only then claim the extraction of the low energy constants reliable. That is very difficult and very CPU-consuming, so that the question of reliable chiral extrapolation will remain a very hot research topic in the lattice community for quite some time.

2.3 Fuss about the Roper resonance

The experimental phenomenon for which the hadron physics phenomenologists do not have a viable explanation is the lightness of the radially excited state with quantum numbers of the nucleon ($J^P = \frac{1}{2}^+$), also known as Roper resonance. The puzzle is that its mass, $m_{N'} \approx 1.44$ GeV, is smaller than that of the first orbital excitation, $m_{\tilde{N}} \approx 1.535$ GeV, which contradicts most of the mass formulae derived by various forms of constituent quark model which suggest $m_N(\frac{1}{2}^+) < m_{\tilde{N}}(\frac{1}{2}^-) < m_{N'}(\frac{1}{2}^+)$. That issue attracted quite a bit of attention recently since some lattice QCD simulations claimed to have found the solution to that puzzle. A special care should be devoted to choosing a good interpolating field, i.e. the one that allows a good overlap with the orbital excitation $J^P = \frac{1}{2}^-$ (also known as S_{11}) as discussed in [21]. The results of that paper, along with those provided by other lattice groups [22], are shown in Fig. 4. In the region of quark masses directly accessed from the lattice studies, the mass pattern is consistent with quark models, i.e., $m_N(\frac{1}{2}^+)$

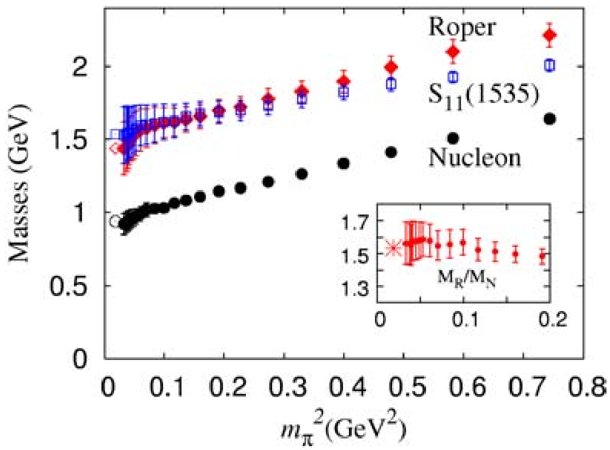

Fig. 5. Results of [23]. Plot similar to those shown in Fig. 4

$< m_{\tilde N}(\frac{1}{2}^-) < m_{N'}(\frac{1}{2}^+)$, although the mass difference, $m_{N'}(\frac{1}{2}^+) - m_{\tilde N}(\frac{1}{2}^-)$, appears to be smaller as the quark mass is lowered. If naively extrapolated to the chiral limit, the level crossing can be envisaged too, i.e. after extrapolation $m_{N'}(\frac{1}{2}^+) - m_{\tilde N}(\frac{1}{2}^-)$ may become negative. That is what [23] claimed to see from their lattice study (see Fig. 5). However, as we saw with the nucleon mass, one should make sure that the finite volume effects and the chiral behavior is well under control. While for the nucleon mass the help in that respect comes from ChPT, there is no such effective theory that provides a similar help for the nucleon excitations. For those reasons I personally think that one should make a better control over various sources of systematic uncertainties in the lattice study of excited nucleons before claiming to observe the level crossing between the nucleon's orbital and radial excitations. This is the point at which one should mention that the clean extraction of the mass of radially excited state has been for a long time a subject to controversies in the lattice community. A recent proposal of [24] provides a possible remedy. The idea is simple and it consists to consider the standard correlation function

$$G(t) = \langle \sum_{\mathbf{x}} J(x) J^\dagger(0) \rangle = \sum_{i\geq 0}^{\infty} Z_i^2 e^{-m_i t}. \qquad (6)$$

We need not only to disentangle the excitations from the leading/dominant contribution (i.e. the one to the ground state), but – of all excitations– we want to isolate the piece corresponding to the first radial excitation only. The proposal of [24] is to consider

$$\begin{aligned}\hat G(t) &= G(t+1)G(t-1) - G(t)G(t) \\ &= 2\sum_{j>i=0}^{\infty} Z_i Z_j e^{-(m_i+m_j)t}\,, \end{aligned} \qquad (7)$$

so that for large time separation the ground state contributes less, and the first radial excitation is then more accessible. The first numerical studies also seem to be encouraging in that respect.

3 Generalized parton distributions (GPD)

Lattice QCD also offers the possibility to study the matrix elements of the local operators sandwiched by the hadron states. This is particularly important for the studies of the CP-violation in the Standard Model. For the recent review on that topic, see [25].

A particularly interesting case to the nuclear physics community is the possibility to get some information about the GPD's of the nucleon. In particular, the matrix elements needed to study the 1^{st} moment have been studied by two lattice groups [26,27].

$$\begin{aligned}\langle P'|O^q_{\{\mu\nu\}}|P\rangle &\equiv \frac{i}{2}\langle P'|\bar q \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} q|P\rangle \\ &= A_2^q(\Delta^2)\bar u(p')\gamma_{\{\mu}\bar p_{\nu\}} u(p) \\ &\quad - B_2^q(\Delta^2)\frac{i}{2m_N}\bar u(p')\Delta^\alpha \sigma_{\alpha\{\mu}\bar p_{\nu\}}u(p) \\ &\quad + C_2^q(\Delta^2)\frac{1}{m_N}\bar u(p')u(p)\Delta_{\{\mu}\Delta_{\nu\}}, \end{aligned} \qquad (8)$$

where $\Delta = p - p'$, and the operator is traceless and symmetrised over the indices in the curly brackets. The form factors A_2, B_2 and C_2 can be extracted for several kinematic configurations which then allows one to study their Δ^2-dependence. Both groups fit their data ($X = A_2, B_2, C_2$) to a dipole ansatz

$$X(\Delta^2) = \frac{X(0)}{(1 - M_X^2/\Delta^2)^2}\,, \qquad (9)$$

that unfortunately does not provide us with more insight in physical mechanism that governs the Δ^2-dependence. [2] It should be stressed, however, that the calculation of the above matrix elements on the lattice is demanding for many reasons. One of the most involving problems is renormalization of the operators on the lattice containing the covariant derivative. The reason is that at non-zero lattice spacing the Lorentz group in Euclidean space $SO(4)$ is replaced by the group of discrete hypercubic rotations $H(4)$, which additionally complicates the renormalization mixing pattern among various combinations of operators with covariant derivatives.

Particularly interesting physics information from the first lattice QCD studies of GPD's is the fraction of the total angular momentum of the nucleon carried by the valence quarks. The total angular momentum of a quark q in the nucleon can be expressed via [28]

$$J_q = \frac{1}{2}\left[A_2^q(0) + B_2^q(0)\right]. \qquad (10)$$

In [26], the following values have been reported: $A_2^u(0) = 0.40(2)$, $A_2^d(0) = 0.15(1)$, and $B_2^u(0) = 0.33(11)$, $B_2^d(0) = -0.23(8)$, and therefore $J_u = 0.37(6)$ and $J_d = -0.04(4)$.

[2] Resonances in the crossed t-channel are poles in the dispersion relations for these form factors. Apart from the convenient fit formula, no reasonable physical significance could be given to the resulting M_X^{dipole}.

In other words, about 30% of the (quenched) proton's angular momentum is carried by the gluons. The situation in the world with $N_f = 2$ dynamical quarks seems to show that the fraction of the total angular momentum carried by the valence quarks is even smaller. A more definite conclusion on this issue necessitates a more control over the chiral extrapolations that are involved in these calculations.

4 Conclusion

In conclusion I would like to point out the main benefits of the lattice QCD approach:

- Lattice QCD offers a unique possibility to study the physics of hadrons on the basis of the QCD lagrangian only.
- The high statistics numerical simulations of QCD on the lattice have so far been done in the so called quenched approximation. Nowadays more and more studies are made by including the effects of dynamical quarks.
- The methodology to extract the physically interesting information from the data produced on the lattice is developed in the world with heavy pions. It is highly important to extend the range of directly accessible quark masses on the lattice to lighter ones in order to confront the quark mass dependence observed on the lattice with the expressions obtained by means of ChPT.
- Many phenomenologically relevant question in particle physics have been studied by using lattice QCD. If we are to make the precision calculation of hadronic quantities on the lattice, we first need to solve the above mentioned problems. More computing power, better algorithms, more clever physical ideas and the combination of all three aspects are essential in reaching that goal.

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